

4767

Mark Scheme

June 2012

Question		Answer	Marks	Guidance
1	(i)	<p><b>EITHER</b></p> $S_{xy} = \sum xy - \frac{1}{n} \sum x \sum y = 600.41 - \frac{1}{10} \times 113.69 \times 52.81 = 0.01311$ $S_{xx} = \sum x^2 - \frac{1}{n} (\sum x)^2 = 1292.56 - \frac{1}{10} \times 113.69^2 = 0.01839$ $S_{yy} = \sum y^2 - \frac{1}{n} (\sum y)^2 = 278.91 - \frac{1}{10} \times 52.81^2 = 0.02039$ $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{0.01311}{\sqrt{0.01839 \times 0.02039}} = 0.677$ <p><b>OR</b></p> $\text{cov}(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = 600.41/10 - 11.369 \times 5.281 = 0.001311$ $\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(0.01839/10)} = \sqrt{0.001839} = 0.04288$ $\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(0.02039/10)} = \sqrt{0.002039} = 0.04516$ $r = \frac{\text{cov}(x,y)}{\text{rmsd}(x)\text{rmsd}(y)} = \frac{0.001311}{0.04288 \times 0.04516} = 0.677$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p><b>[5]</b></p>	<p>For method for <math>S_{xy}</math></p> <p>For method for at least one of <math>S_{xx}</math> or <math>S_{yy}</math></p> <p>For at least one of <math>S_{xy}</math>, <math>S_{xx}</math> or <math>S_{yy}</math> correct</p> <p>For fully correct structure of <math>r</math></p> <p>For answer rounding to 0.68</p> <p>For method for cov (<math>x,y</math>)</p> <p>For method for at least one msd or rmsd</p> <p>For at least one of cov (<math>x,y</math>), msd or rmsd correct</p> <p>For fully correct structure of <math>r</math></p> <p>For answer rounding to 0.68</p> <p>Methods mixed – max M0M1A1M0A0</p>
1	(ii)	<p><math>H_0: \rho = 0</math>  <math>H_1: \rho \neq 0</math> (two-tailed test)</p> <p>where <math>\rho</math> is the population correlation coefficient</p> <p>For <math>n = 10</math>, 10% critical value = 0.5494</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>For <math>H_0, H_1</math> in symbols. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population correlation coefficient.</p> <p>For defining <math>\rho</math>. Condone omission of “population” if correct notation <math>\rho</math> is used, but if <math>\rho</math> is defined as the <b>sample</b> correlation coefficient then award <b>B0</b>.</p> <p>CAO</p> <p>Note that critical values for a one-tailed test at the 10% level are not available in tables.</p>

4767

## Mark Scheme

June 2012

		<p>Since <math>0.677 &gt; 0.5494</math> the result is significant.</p> <p>(Thus we have sufficient evidence to) reject <math>H_0</math></p> <p>There is sufficient evidence at the 10% level to suggest that there is correlation between times for the first and last sections.</p>	<p>M1</p> <p>A1*</p> <p>E1dep*</p> <p>[6]</p>	<p>For sensible comparison leading to a conclusion provided that <math> r  &lt; 1</math>. The comparison can be in the form of a diagram as long as it is clear and unambiguous. Sensible comparison: e.g. <math>0.677 &gt; 0.5494</math> is ‘sensible’ whereas <math>0.677 &gt; -0.5494</math> is ‘not sensible’. Reversed inequality sign e.g. <math>0.677 &lt; 0.5494</math> etc. gets max M1 A0.</p> <p>For reject <math>H_0</math> o.e. FT their <math>r</math> and critical value from 10% 2-tail column.</p> <p>For correct, non-assertive conclusion in context. Allow ‘<math>x</math> and <math>y</math>’ for context. E0 if <math>H_0</math> and <math>H_1</math> not stated, reversed or mention a value other than zero for <math>\rho</math> in <math>H_0</math>. Do not allow ‘positive correlation’ or ‘association’</p>	
1	(iii)	<p>The underlying population must have a bivariate Normal distribution.</p> <p>The points in the scatter diagram should have a roughly elliptical shape.</p>	<p>B1</p> <p>E1</p> <p>[2]</p>	<p>Condone “bivariate Normal distribution”, “underlying bivariate Normal distribution”, but <b>do not allow</b> “the <b>data</b> have a bivariate Normal distribution”</p> <p>Condone ‘oval’ or suitable diagram</p>	
1	(iv)	<p>The hypothesis test has shown that there appears to be correlation.</p> <p>However it could be that there is a third causal factor</p>	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>For relevant comment relating to the test result or positive value of <math>r</math> in supporting (unless FT leads to not supporting) the commentator’s suggestion. Or correlation does not imply causation. There may be a third factor. For questioning the use of the word ‘must’</p> <p>Allow any two suitable, statistically based comments.</p>	
1	(v)	(A)	<p>B1*</p> <p>E1dep*</p> <p>[2]</p>	<p>B1 for 0.7646 seen</p> <p>E1 for comment consistent with their (ii) provided <math> r  &lt; 1</math></p>	

4767

## Mark Scheme

June 2012

1	(v)	(B)	One advantage of a 1% level is that one is less likely to reject the null hypothesis when it is true. One disadvantage of a 1% level is that one is more likely to accept the null hypothesis when it is false.	E1 E1 [2]	o.e. Wording must be clear. o.e.
2	(i)		Binomial(1200, 1/300)	B1 B1dep [2]	For binomial. For parameters Allow B(1200, 1/300) and B(1200, 0.00333)
2	(ii)		Because $n$ is large and $p$ is small	E1, E1 [2]	Allow $n$ is large and $np < 10$ . Allow “sample is large” for $n$ is large and “mean $\approx$ variance” for “ $p$ is small”
2	(iii)		$\lambda = 1200 \times 1/300 = 4$ (A) $P(X = 1) = e^{-4} \frac{4^1}{1!} = 0.0733$ (3 s.f.) or from tables $= 0.0916 - 0.0183 = 0.0733$ (B) Using tables: $P(X > 4) = 1 - P(X \leq 4)$ $= 1 - 0.6288 = 0.3712$	B1 M1 A1 M1 A1 [5]	For $\lambda$ FT their $p$ For attempt to find $P(X = 1)$ using Poisson p.d.f. or tables Allow answers which round to 0.073 www. FT their $\lambda (= np)$ . No FT for $\lambda = 1/300$ . For finding $1 - P(X \leq 4)$ CAO For answers rounding to 0.371 www
2	(iv)		$\mu = 80$ $\sigma^2 = 80$	B1 B1 [2]	If symbols/words used then they must be correct. Allow $\sigma^2$ rounding to 79.7 from original binomial. FT their $\lambda (= np)$
2	(v)	(A)	$P(Y \geq 90) = P\left(Z \geq \frac{89.5 - 80}{\sqrt{80}}\right)$ $= P(Z > 1.062) = 1 - \Phi(1.062)$ $= 1 - 0.8559 = 0.1441$	B1 M1 A1cao [3]	For correct continuity correction. For probability using correct tail and structure (condone omission of c.c.) $\sigma^2 = 79.73$ leads to $P(Z > 1.064)$ $\sigma^2 = 79.73$ leads to $1 - 0.8563 = 0.1437$ . Allow 0.144 www. NOTE 0.1441 from B(24000, 1/300) gets 0/3

4767

## Mark Scheme

June 2012

2	(v)	(B)	$P(Y \leq k) > 0.05$ From tables $\Phi^{-1}(0.05) = -1.645$ $\frac{(k + 0.5) - 80}{\sqrt{80}} = -1.645$  $k + 0.5 = 80 - (1.645 \times \sqrt{80}) = 65.29$ $k > 64.79$  So least value of $k = 65$	B1 M1   A1   A1  <b>[4]</b>	For $\pm 1.645$ seen For correct equation in $k$ seen or equivalent – e.g. allow +1.645 used if numerator reversed. FT their $\mu$ , $\sigma^2$ and z-value. Condone omission of, or incorrect, continuity correction.  A1 for 65.29 or 64.79 or 65.79 ( $\sigma^2 = 79.73$ leads to 65.31 or 64.81 or 65.81) Allow 3s.f.  For rounding <b>64.79 or 64.81 up to give <math>k = 65</math></b> . See additional notes for alternative method
3	(i)		$P(X \geq 750) = P\left(Z \geq \frac{750 - 751.4}{2.5}\right)$ $= P(Z > -0.56) = \Phi(0.56) = 0.7123$	M1 M1  A1  <b>[3]</b>	For standardizing For correct structure (M0 if continuity correction used)  CAO Allow 0.712 www
3	(ii)		$P(\text{all 6 at least 750ml}) = 0.7123^6$ $= 0.1306$	M1 A1 <b>[2]</b>	For (their answer to part (i)) <sup>6</sup> FT 3s.f.
3	(iii)		$P(Y=0) = \binom{25}{0} \times 0.8694^{25} (= 0.0302)$ $P(Y=1) = \binom{25}{1} \times 0.8694^{24} \times 0.1306 (= 0.1135)$ $P(Y=0) + P(Y=1) = 0.144$ $P(Y \geq 2) = 1 - 0.144$ $= 0.856$	M1  M1  M1dep A1 <b>[4]</b>	For using Binomial(25, $p$ ) with their $p$ from part (ii)  For correct structure of either $P(Y = 0)$ or $P(Y = 1)$ with their $p$ from part (ii) M0 if $p$ and $q$ reversed  For 1 – sum of both probabilities CAO

4767

## Mark Scheme

June 2012

3	(iv)	$P\left(Z < \frac{750 - \mu}{2.5}\right) = 0.02$ $\Phi^{-1}(0.02) = -2.054$ $\frac{750 - \mu}{2.5} = -2.054$ $\mu = 750 + 2.054 \times 2.5$ $= 755.1$	B1 M1 M1 A1 <b>[4]</b>	For $\pm 2.054$ seen. Allow $\pm 2.055$ For correct equation as seen or equivalent. FT $\sigma = \sqrt{2.5}$ . M0 if c.c. used. For correctly rearranging their equation (if 750 used in numerator) for $\mu$ , FT their $z$ cao Condone 755 or 5 s.f. rounding to 755.1 www	
3	(v)	$P\left(Z < \frac{750 - 751.4}{\sigma}\right) = 0.02$ $\frac{750 - 751.4}{\sigma} = -2.054$ $\sigma = \frac{-1.4}{-2.054}$ $= 0.682$	M1 M1 A1 <b>[3]</b>	For correct equation as seen or equivalent For correctly rearranging their equation (if 750 used in numerator) for $\sigma$ unless this leads to $\sigma < 0$ cao Allow answers rounding to 0.68 www	
3	(vi)	Probably easier to change the mean (as reducing the standard deviation would require a much more accurate filling process). However increasing the mean would result in fewer bottles being filled overall and so less profit for the owners, so <b>reducing the standard deviation</b> would be <b>preferable</b> to the vineyard owners.	E1 E1 <b>[2]</b>	For “preferable to reduce the standard deviation” with valid reason.	
4	(a)	(i)	Expected frequency = $67/150 \times 57 = 25.46$ Contribution = $(34 - 25.46)^2 / 25.46$ $= 2.8646$	B1 M1 A1 <b>[3]</b>	For 25.46 For valid attempt at $(O-E)^2/E$ Correct values used to give answer which rounds to 2.8646 <b>NB Answer given</b>

4767

## Mark Scheme

June 2012

4	(a)	(ii) $H_0$ : no association between type of cake and classification of person. $H_1$ : some association between type of cake and classification of person. Test statistic $X^2 = 12.86$ Refer to $X^2_3$ Critical value at 1% level = 11.34 Result is significant There is evidence to suggest association between type of cake and classification of person. NB if $H_0$ $H_1$ reversed, omitted or ‘correlation’ mentioned, do not award first B1 or final E1	B1  B1 B1 B1 E1  [5]	For both hypotheses in context  For 3 degrees of freedom CAO For cv. No FT from here if wrong/omitted For significant For correct, non-assertive conclusion, in context.	
4	(b)	$\bar{x} = 4.995$ $H_0: \mu = 5$ $H_1: \mu < 5$  Where $\mu$ denotes the <b>mean</b> content of the bags of flour (in the population)  $\text{Test statistic} = \frac{4.995 - 5.0}{0.0072 / \sqrt{8}} = \frac{-0.005}{0.002546} = -1.964$ Lower 5% level 1 tailed critical value of $z = -1.645$  $-1.964 < -1.645$ so significant.  There is sufficient evidence to reject $H_0$ There is sufficient evidence to suggest that the average contents of bags is less than 5kg.	B1 B1 B1  B1  M1* A1  B1*  M1 dep*  A1  [9]	For 4.995 seen For use of 5 in hypotheses. For both correct. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population mean. For definition of $\mu$ in context. Condone omission of “population” if correct notation $\mu$ is used, but if $\mu$ is defined as the <b>sample</b> mean then award <b>B0</b> .  must include $\sqrt{8}$ FT their $\bar{x}$ . Allow +1.964 only if later compared with +1.645  For -1.645 No FT from here if wrong. Must be -1.645 unless it is clear that absolute values are being used. For sensible comparison with correct c.v. leading to a conclusion.  For non-assertive conclusion in words and in context. No FT here. See additional notes.	

4767

Mark Scheme

June 2012

ADDITIONAL NOTES REGARDING QUESTION 2 (v) B

M1 for using a trial and improvement method with  $N(80,80)$  or  $N(80, 79.73)$  to find  $P(Y \leq k)$  for any  $k$ . The distribution being used needs to be made clear.

A1 for  $P(Y \leq 66) = 0.0587\dots$  (0.0584... from  $\sigma^2 = 79.73$ ) or  $P(Y \leq 65) = 0.0467\dots$  (0.0464... from  $\sigma^2 = 79.73$ )

A1 for both

Final A1 not available if 66 and 65 used

Or

A1 for  $P(Y \leq 65.5) = 0.0524\dots$  (0.0521... from  $\sigma^2 = 79.73$ ) or  $P(Y \leq 64.5) = 0.0415\dots$  (0.0412... from  $\sigma^2 = 79.73$ )

A1 for both

A1 for least value of  $k = 65$ , dependent on previous two A marks earned.

ADDITIONAL NOTES REGARDING QUESTION 4 (b)Critical Value Method

$5 - 1.645 \times 0.0072 \div \sqrt{8}$  gets M1\*B1\*

= 4.9958... gets A1

$4.995 < 4.99581\dots$  gets M1dep\* for sensible comparison

A1 still available for correct conclusion in words & context

“Confidence Interval” Method

$4.995 + 1.645 \times 0.0072 \div \sqrt{8}$  gets M1\* B1\*

= 4.9991.. gets A1

NOTE that the final M1dep\* A1 available only if 1.645 used.

$5 > 4.9991\dots$  gets M1

A1 still available for correct conclusion in words & context

Probability Method

Finding  $P(\text{sample mean} < 4.995) = 0.0248$  gets M1\* A1 B1

$0.0248 < 0.05^*$  gets M1dep\* for a sensible comparison if a conclusion is made.

A1 available for a correct conclusion in words & context.

Condone  $P(\text{sample mean} > 4.995) = 0.9752$  for M1 but only allow A1 B1 if later compared with 0.95, at which point the final M1 and A1 are still available

ADDITIONAL NOTE REGARDING OVER-SPECIFICATION OF ANSWERS

Over-specification by providing final answers correct to 5 or more significant figures will be penalised. When this applies, candidates may lose no more than 2 marks per question and no more than 4 marks in total. The only exception to this rule is in Question 3 part (iv) – see guidance note.